IDENTIFICATION AND MODELLING OF LINEAR DYNAMIC SYSTEMS

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Summary System identification and modelling are very important parts of system control theory. System control is only as good as good is created model of system. So this article deals with identification and modelling problems. There are simple classification and evolution of identification methods, and then the modelling problem is described. Rest of paper is devoted to two most known and used models of linear dynamic systems.

1. INTRODUCTION

Effective system control with using of continuous-time or discrete-time control technology assumes accomplishment of two basic preconditions. The first is needed to know controlled system as good as possible. This means making of suitable mathematical model for this system. The second step is voting or creating of adequate control algorithm, or setting of control law parameters of fixed structure, which is predetermined. The synthesis depends on attributes and structure of mathematical model both of these cases.

Mathematical model is possible to make dual ways. The first approach requires using of analytic methods. This means model making based on physical, chemical or other characteristic processes that are running in system. These models have some disadvantages. They cannot include all real factors and they can give non-linear functionalities. The second way is application of experimental identification methods. These methods can process data measured on concrete system [1].

2. CLASIFICATION OF IDENTIFICATION METHODS

The purpose of experimental identification is specification of controlled object's mathematical model. This model can by defined by differential equation for continuous-time systems or difference equation for discrete-time systems. There are more possibilities how to classify identification methods.

Identification processes is possible to sort by character of identification experiment:

- active experiment identification,
- passive experiment identification.

The first type is possible to realise during routine system working. The second type needs special testing signal manipulations. Input testing signal can be step change of constant signal (or unit jump), defined impulse, or harmonic signal. Passive experiment identification enables evaluation of general non-standardized signal (progressive integration method and convolution integral methods) or there is evaluated stochastic signal with defined parameters (correlative methods) [2].

By mathematical apparatus, which is needed for evaluation, is possible to separate identification methods into this groups:

- deterministic,
- stochastic,
- statistic.

Deterministic methods evaluate responses for special signals (integrative methods). The least squares method (including family of her modifications) and maximal credibility method comes under category of statistic methods. Wiener – Hopf's equation's solving represents stochastic approach.

By the method of computing tools utilize in system identification process is possible this identification method's division:

- methods for manual processing,
- methods for one-off processing,
- running identification methods.

Methods for manual processing are based on subjective rating of benchmarks so they are not suitable for computer calculation.

Methods for one-off processing apply some of introduced mathematical apparatus for measured data file. The result of this calculation is mathematical model represented by continuous or discrete transmission. These methods are suitable for off-line using of computer technology.

Running identification methods (also called recursive) process sampled values of input and output quantities. They make new actualised identification parameter's guess in each sampling period by iterative method. They are suitable for on-line service of computer technology.

3. EVOLUTION OF IDENTIFICATION METHODS

Identification of continuous-time dynamic systems concentrated to two goals to the first half of sixties. The first goal was evolution of methods, what could evaluate transitional characteristics, or some other responses for normalized signals. The second goal was evolution of system identification, when input signal of system was stochastic. This identification was based on Wiener – Hopf's equation's solving. Both groups come under category of methods for one-off processing.

Next evolution of identification methods was connected with computer technology progress. There were analogue computers, but digital computers were start multiplying in that time. Analogue computer could use continuous-time methods, but digital computer could use discrete-time methods. Continuous-time or discrete-time method could make continuous-time or discrete-time model of system. So system identification could be realised with using one of these four modes:

- continuous-time transit and method,
- continuous-time transit, discrete-time method,
- discrete-time transit, continuous-time method,
- discrete-time transit and method.

The first combination is analogue method; the last fourth combination is digital method. Other two combinations are connected with hybrid methods. Digital methods broke into branch together with digital computer technology start in seventies. This trend is sustentative to these days.

The base of digital methods is one-off processing method for determining of identification parameter's vector Θ. Least mean squares method or maximal credibility method is usable for it. Then running identification methods are extrapolated from one-off processing methods. Centre of a running identification problem is based on iteration of identification parameter's vector's calculation. That means repeating of calculation with every another sampling rate period, when some new input and output data was acquired from the system (new identification parameter's vector's calculation).

By the type of model, method of his deducing and identification principle selecting is possible to differentiate several types of running identification methods:

a) running least mean squares method,
b) running extended least mean squares method,
c) running generalized least mean squares method,
d) running maximal credibility method,
e) running method of auxiliary variables,
f) method with root square decomposition of covariance matrix (*REFIL*),
g) method with *LD* decomposition of covariance matrix (*LDFIL*).

4. MODELING OF LINEAR DYNAMIC SYSTEMS

Creating of system's mathematical model is quest to find functionality:

$$y = f[u, v], \tag{1}$$

where y is output system's quantity, u is input system's quantity and v is a representative quantity of error influences, what affect on the system [5].

Searched function characterise system's operation, which is defined with her response *y*. This response is reaction for input quantities and some other measurable quantities, what can affect on system's function (error quantities *v*).

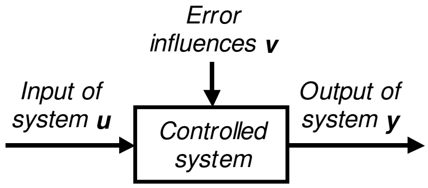

Fig.1 Modelled system

But deterministic system's response on input quantities is rarity in practice. Non-measurable influences, what are causes of this fact we usually name stochastic effects. With respect to this consideration is possible to develop defined model of system (1) to new form:

$$y = f[u, v] + n, \tag{2}$$

where *n* is representative of stochastic influences.

Model making of continuous-time dynamical system need manipulation with derivative of measured signal. Fortunately, discrete-time model making of system is much more easier. Signals are acquired in periodical time intervals, what are defined by sample rate *T*. On basis of dependence, described with formula (2), is possible to create this general description of dynamical system:

$$\begin{aligned} y(k) = f[\,&y(k-1), \ldots, y(k-na), \\ &u(k-1), \ldots, u(k-nb), \\ &v(k-1), \ldots, v(k-nd), k] + n(k), \end{aligned} \tag{3}$$

where $y(k)$ for $t = kT$ is value of output quantity.

Determination of variable $n(k)$, which represent stochastic influences in model, is problematic. There is one possibility, how to model error $n(k)$. It is possible to represent it by some well-known signal. We can acquire this signal like output of filter, which we can use for filtering of known attributes noise. So attributes of error are the same than attributes of her filter. Then in a similar way that was defined system's description (3), we can define new description of system with respect of noise modeling filter attendance:

$$\begin{aligned} y(k) = f[\,&y(k-1), \ldots, y(k-na), \\ &u(k-1), \ldots, u(k-nb), \\ &v(k-1), \ldots, v(k-nd), \\ &e_s(k), e_s(k-1), \ldots, e_s(k-nc), k]. \end{aligned} \tag{4}$$

In this formula is problematic random part of vector e_s, exactly element $e_s(k)$. This element of vector is not measurable from time aspect, because output of system *y* can be only function of previous values of measured quantities. The most nearest value of vector *e*, which correspond with this demand, is value $e_s(k-1)$.

5. ARMAX MODELS

Let is the system defined by linear difference equation, which is defined:

$$
\begin{aligned}
& a_n y[(k+n)T] + a_{n-1} y[(k+n-1)T] + \ldots \\
& \ldots + a_1 y[(k+1)T] + a_0 y(kT) = \\
& = b_m u[(k+m)T] + b_{m-1} u[(k+m-1)T] + \ldots \\
& \ldots + b_1 u[(k+1)T] + b_0 u(kT) ,
\end{aligned} \tag{5}
$$

where a_i, b_j are constant coefficients of equation, $u(kT) \equiv u(k)$ is input quantity, $y(kT) \equiv y(k)$ is output quantity, $m \le n$ is requirement of system's physical viability and $y(0), \ldots, y[(n-1)T]$; $u(0), \ldots, u[(n-1)T]$ are initial conditions known in advance, what are needful for this equation's solution [6]. Let is this equation arranged for represent $y(k)$, with considering of error and noise component. If the function *f* is linear, then we can formulate the equation (4) in this form:

$$
\begin{aligned}
y(k) = & -\sum_{i=1}^{na} a_i y(k-i) + \sum_{i=1}^{nb} b_i u(k-i) + \\
& + \sum_{i=1}^{nd} a_i v(k-i) + e_s(k) + \sum_{i=1}^{nc} c_i e_s(k-i) .
\end{aligned} \tag{6}
$$

More often this formula can be defined with help of Z-transform, with using of time-delay operator:

$$A(z^{-1}) y = B(z^{-1}) u + D(z^{-1}) v + C(z^{-1}) e_s , \tag{7}$$

where individual polynomials are:

$$
\begin{aligned}
A(z^{-1}) &= 1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_{na} z^{-na} , \\
B(z^{-1}) &= b_1 z^{-1} + b_2 z^{-2} + \ldots + b_{nb} z^{-nb} , \\
C(z^{-1}) &= 1 + c_1 z^{-1} + c_2 z^{-2} + \ldots + c_{nc} z^{-nc} , \\
D(z^{-1}) &= d_1 z^{-1} + d_2 z^{-2} + \ldots + d_{nd} z^{-nd} .
\end{aligned} \tag{8}
$$

Both types of notations (6) and (7), are notations of linear dynamic system's model, called *ARMAX* (*Auto Regressive Moving Average model with eXternal input*) (Fig.2) [2].

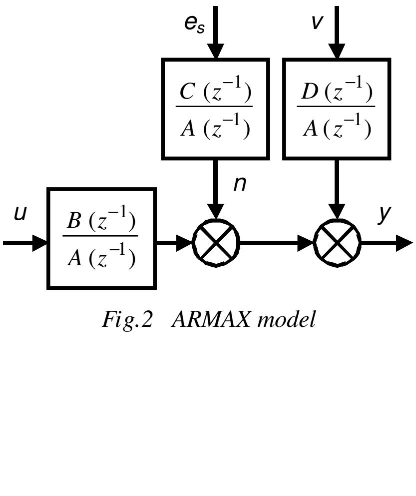

Fig.2 ARMAX model

6. ARX MODELS

Model, which was deduced in previous chapter, is connected with one problem. If we will need to determine coefficients of polynomials *A*, *B*, *C*, *D*, we can meet with opposition of C-polynomial coefficients specifies. These problems will give rise to fictionally noise $e_s(k)$, which is not quantifiable. In these situations is better to use next variant of model, which come out of *ARMAX* model. This type of model is easier, and it is defined than:

$$
\begin{aligned}
y(k) = & -\sum_{i=1}^{na} a_i y(k-i) + \sum_{i=1}^{nb} b_i u(k-i) + \\
& + \sum_{i=1}^{nd} a_i v(k-i) + e_s(k) ,
\end{aligned} \tag{9}
$$

or with help of Z-transform, with using of time-delay operator:

$$A(z^{-1}) y = B(z^{-1}) u + D(z^{-1}) v + e_s . \tag{10}$$

This easier model of linear dynamic system is called as *ARX* (*Auto Regressive model with eXternal input*), or model with error in equation too (Fig.3).

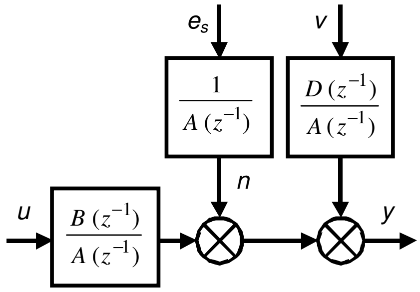

Fig.3 ARX model

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